Ability in a multi-agent context: a model in the Situation Calculus

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Abstract. This paper studies the notion of ability and its relation with the notion of action in a multi-agent context. It introduces the distinction between two notions respectively called "theoretical ability" and "ability". The main contribution of this paper is a model of these notions in the Situation Calculus.

1 Introduction

Allocating tasks or planning in a multi-agent context [8], [3], requires taking into account what the agents are able to do, i.e., the agent abilities, in order to assign tasks to agents who are able to perform them.

The notion of ability must then be modelled and this implies to explicit the parameters which define this notion.

Obviously, the agent itself, or the group of agents, is one of these parameters. But what is the nature of what the ability applies on? For instance, when we say that *John is able to paint the door*, do we mean that John is able to perform a particular action which consists in applying paint with a specific brush on the door? Or do we mean that John is able to see to it that the door is painted, by the means he wants, for instance by delegating this task to someone else? Modelling ability thus implies modelling actions.

In the literature, there are two main approaches to action theory. The first one consists in giving in the language means to explicitly represent actions. This is the case of dynamic logic for instance [9], which offers modal operators to speak about the execution of an action, and also the execution of an action by an agent. This is also the case of situation calculus [14, 16], which allows one to represent actions, their preconditions and their effects, but also situations, considered as results of the successive application of actions in an initial situation. On the contrary, actions are not explicitly represented in the second approach. The operators defined there only allow to express the fact that the agent sees about some property to be true (cf. the operator *stit* [11] and the notion of *agency* in [7]).

As the notion of ability is strongly linked to the notion of action, it has been studied according to these two approaches. For instance, the multi modal dynamic logic KARO [18] aims at defining agent ability to perform an action according to the first approach. Primitive concepts are the agent's knowledge, its capacity to perform an action, the effects of an action and the opportunity associated with an action. Ability and opportunity are two intertwined notions, we will come back on it later.

Concerning the second approach, the notion of ability does not bear on actions, but on the fact that a property is true [10, 7]. These two formalisms are based on propositional modal logics. In [7], Elgesem defines ability and action as primitive notions. He considers a function f which determines for a given world w and a goal φ the worlds in which the agent has realized its ability to see to it that φ is true from w. Thus, an agent is able to see to it that φ is true if and only if the set of worlds $f(w, \varphi)$ is not empty. With this definition, ability and action, which is also defined by f, are two binded notions. For instance, if an agent sees to it that φ is true, then this agent is able to see to it that φ. In [10], Horty uses temporal models to represent actions: an agent sees to it that φ is true at a moment m if it restricts the "histories" which m belongs to in order that φ is true. The ability for an agent to see to it that φ is true is defined as the possibility (in the classical sense, see [2]) for the agent to see to it that φ is true. Let us notice that with this formalism, Horty avoids several paradoxes. In particular, it cannot be deduced that if φ is true, then the agent has the ability to see to it that φ.

We must also mention [12], in which the authors use the situation calculus to model the notion of "ability to reach a goal", i.e., "ability to make a proposition true". Two definitions are given in a mono-agent context. According to the one the authors find the simplest to use, the agent has the ability, in a situation s, to make φ true (i.e., the agent is able to reach the goal φ) if there exists a sequence of actions such that the agent knows in s that executing these actions will make φ true. In other terms, the agent has the ability to make φ true if he knows a plan to achieve φ.

This brief state of the art shows that there is no consensus on what the ability applies on. However, we find in the literature several points of agreement relatively to the notion of ability.

First, the notion of ability must not be confused with the notion of possibility nor with the notion of permission [17]. These possible confusions are due to ambiguities of the natural language. For instance, the sentence "I can open the door" is sometimes used to say "I am able to open the door" according to the notion of ability we study here. But this sentence is also sometimes used to say "I have now the possibility to open the door" (because, for instance, the door is now unlocked), but this does not mean that I am able to do so. Here, it refers to a notion of possibility. Finally, this sentence is sometimes used to mean "I have the permission to open the door", which still does not imply that I am able to do so and which refers here to a deontic notion.

Secondly, several people agree on the fact that two kinds of ability must be distinguished [17, 18, 5, 1].

One can first distinguish what is called "generic ability" by some people (or "ability"by others) and which refers to the agent's competences to perform an action in normal conditions. "I can open the door" means here that I know what to do to open the door, independently of my current intellectual or physical state and of the current state of the world. Thus, with this point of view, I can say "I am able to open the door" even if my arms are broken or the door is locked.

One can also distinguish what is called "occasional ability" by some people (or "pragmatic possibility", or "opportunity to exercise an ability" by others) and which refers to the current situation. Here, "I can open the door" means that I have the generic ability to open the door *and* the current situation is such that conditions are favourable for me to use this generic ability (for instance, my arms are not broken and the door is unlocked).

Finally, in a multi-agent context, one of the main problems is to define the notion of ability relatively to a group of agents, and in particular to infer the ability of the group from the abilities of the individuals of the group. For instance, what are the conditions for saying that a group of people is able to paint the door? Or a group of people is able to first sand the door, then to paint it? The problem is not trivial since the notion of ability previously called "occasional" is a dynamic notion which depends on the state of the world in which it is evaluated. But in a multi-agent context, the dynamic of the world is hard to foresee because several agents may change the world.

This paper presents a preliminary study of the notion of ability in a multi-agent context. As far as we know, no previous work has already attacked the same question in a multi-agent context. In particular, it must be noticed that the notion we are trying to model here is different from the one Pauly has studied [15]. Indeed, in his work "an agent (or a group of agents) can bring about a proposition" means that this agent (or group of agents) has a (collective) efficient strategy which makes this proposition true, whatever the other people do. In particular, the logic defined by Pauly does not apply when a first agent can bring about a proposition and another one can bring about its contrary.

This paper is organized as follows. In section 2, we informally discuss some requirements about the notion of ability. In section 3 we propose a formal model, in the Situation Calculus, of concepts related to ability and we justify the use of such formalism. Some properties of this model are given in 4, and an example is detailed in 5. Section 6 presents an implementation of our modelling and section 7 is devoted to a discussion and presents some perspectives.

2 Informal requirements about ability

Our modelling lies on the following choices:

- The ability we focus on bears on actions. We aim at characterizing the meaning of being able to perform actions, i.e., to perform a procedure [6].
- We aim at explicitly representing actions, their preconditions and their effects. We consider the general case when some primitive actions may require several agents to be performed (for instance, lifting an heavy door requires two agents).
- We aim at defining the ability of an agent to perform an action as the combination of its competences and some favourable conditions that allow it

to perform that action. But we also aim at making a difference between the conditions which are related to the agent only from those which are not. For doing so, we introduce an intermediary notion called "theoretical ability".

2.1 Model of action

In our model of action, any primitive action is specified by its preconditions which are the conditions under which it can be performed, independently of any agent. When these conditions are true, we say that *the action is possible* [16].

Example 1. For instance, "painting" is only possible if there is some paint and a brush.

2.2 Model of ability

In our model, the primitive notion is the notion of competence described as follows:

Competence Competence represents the knowing-how of the agent (or agents) relatively to an action. This knowledge may be inborn or may result from a learning phase. In our model, this information is considered as *primitive*.

For instance, we will have initial data like: "John is competent to paint a door" or "John and Peter are competent together to lift the door". The first sentence means that John knows the successive gestures he has to make so that the door is painted. The second sentence means that they both know how to coordinate their gestures in order to lift the door.

In this work, we will assume that the competences of the agents can not be deleted: once an agent is competent to perform an action, he will always be. This assumption seems to be justified in many applications with rather short temporal horizon where it can be assumed that the agents do not loose their competences. As we will see, this assumption can be easily removed.

It must be noticed that this notion of competence is different from the one Cohen and Levesque consider in [4], where, if an agent competent for a proposition p believes p, then p is true.

Theoretical ability From the notion of competence,we first define the notion of theoretical ability as follows:

Definition 1. *Let A be a non empty group of agents (possibly a singleton) and α be a primitive action. A is theoretically able to perform α if:*

1. *A is competent to perform α*
2. *some conditions, related to the agents of A, are true.*

Remember that competence is considered to be a primitive notion. The conditions expressed in point 2 concern the agent (its physical state for instance), but not all the environment.

Example 2. For instance, an agent is theoretically able to paint a door if it is competent for paint a door and if it is not tired.

Notice that this notion is close to the notion of *ability* of [18].

Ability The notion of ability is finally defined from the notion of theoretical ability by taking into account the conditions which define the possibility to perform the action.

Definition 2. *Let A be a non empty group of agents (possibly a singleton) and α be a primitive action. A is able to perform α if:*

1. *A is theoretically able to perform α*
2. *α is possible.*

Notice that the notion of possibility in this definition is the one defined by Reiter in [16]. The possibility here is a set of conditions concerning the state of the world excepting the agent.

Example 3. For instance, an agent is able to paint a door if it is theoretically able to paint the door (i.e, competent for painting the door, not tired) and if there is some paint and a brush.

This notion of ability is a kind of occasional ability in the sense of section 1.

It can also be noticed that the previous condition "α is possible" is very close to the notion of "opportunity to exercise an ability" mentioned in [5], as well as the one mentioned in [18]. It means that, in our approach, an agent is able to perform an action if it has the opportunity to exercise the theoretical ability to perform this action.

2.3 Extensions to more complex actions

Considering only primitive actions is not enough and we must also consider more complex actions obtained by composition of primitive ones. In this preliminary work, we focus on sequences (like for instance: "lift the door, then paint it".)

We would like to validate the following assertion (cf. [18]): agent a is able to perform the sequence α then β if a is able to perform α and, once a has performed α, a is able to perform β.

Example 4. For instance, assume that sanding a door is tiring. Then, we would like to deduce that John is not able to sand then to paint the door (i.e, not able to perform the action "sand then paint"). Indeed, even if John is able to sand the door, once he will have sanded it, he will be tired. Thus, he will not be able to paint the door.

2.4 Deriving ability of a group from abilities of individuals

In a multi-agent context, it is necessary to extend these previous notions for a group of agents. We will say that a group of agents is able to perform a primitive action if one of its sub-group (possibly a singleton) is able to perform it. Notice that in some cases, only a subgroup may be competent to do some action: an agent does not have the competence to carry a piano, but a group of three agents may have it.

As for the sequences, we would like to validate the fact that a group is able to perform the action "α then β" if one of its sub-group is able to perform α and, once this sub-group has performed α, the group is able to perform β.

Example 5. Consider now that Peter is also competent to paint the door and is not tired. Then the group {John, Peter} is able to sand the door then to paint it. Indeed, John is able to sand it (see previously) and, once John has sanded the door, the group is still able to paint the door (because Peter has remained not tired).

3 Model of ability in the Situation Calculus

3.1 The Situation Calculus

We suggest to use the Situation Calculus to model these notions for two reasons:

- firstly, this formalism is a good candidate for modelling actions since it offers means to explicitly express preconditions and effects of actions;
- secondly, an important problem underlying this present work, the frame problem (i.e, how to express what are the changings induced by the performance of an action by an agent and how to express what remains unchanged), has been provided a solution in the Situation Calculus by Reiter.

3.2 The language

We consider a first order language $\mathcal{L}_{CS}$ which will allow us to model and reason about actions and ability. In this language, the changes of the world are resulting from action performances. It is defined as follows:

- a set of constants to represent agents A.
- a set of functions and constants used to represent primitive actions, with parameters or without.
  For instance the term "paint(x)" will represent the action "to paint the object x".
- A unary predicate $primitive(.)$ used to list the primitive actions.
  Thus, $primitive(paint(x))$ means that $paint(x)$ is a primitive action.
- a binary function ; used to represent the sequence of actions.
  $sand(x); paint(x)$ will represent the action which consists in sanding the object x then painting it.

- a constant S_0 used to represent the initial situation.
- a ternary function *do*.
  $do(\mathcal{A}, paint(x), s)$ represents the situation which follows from the situation s, when the group of agents $\mathcal{A}$ has painted the object x.
  Notice that here, unlike the "classical" Situation Calculus, the agent is not a parameter of the function which represents the action, but is a parameter of the function *do* which represents the performance of the action.
- a set of predicates called relational *fluents* which represent properties which may be changed by the performance of an action. The last argument of a fluent is a situation.
  For instance, $painted(door, S_0)$ expresses that the door is painted in the initial situation S_0.
- a particular binary *fluent Poss* used to express that an action is possible in a situation.
- a particular binary *fluent* is *competent* and is used to represent the fact that an agent (or a group) is competent for performing a primitive action.
  For instance, $competent(\{a\}, paint(door), s)$ expresses that agent a is competent to paint the door in situation s.
- a particular ternary *fluent* is *able_t* and is used to represent the fact that an agent (or a group) is theoretically able to perform an action.
  $able\_t(\{a\}, paint(door), S_0)$ expresses that agent a is, in situation S_0, theoretically able to paint the door.
- a particular ternary *fluent* is *able* and is used to represent the fact that an agent (or a group) is able to perform an action.
  $able(\{a\}, paint(door), S_0)$ expresses that agent a is, in situation S_0, able to paint the door.

3.3 The axioms

Description of the initial situation First, the initial state of the world must be represented. For doing so, for any fluent f and for any tuples $t_1, \ldots, t_n$ of ground terms such that $f(t_1, \ldots, t_n)$ is true in the initial situation, we consider the following axiom:

$$f(t_1, \ldots, t_n, S_0) \tag{1}$$

In particular, since *competent* is a fluent, for any group G competent for performing the primitive action α in the initial situation S_0, we consider the following axiom:

$$competent(G, \alpha, S_0) \tag{2}$$

Primitive actions For any primitive action α, we consider an axiom of the following form:

$$primitive(\alpha) \tag{3}$$

Precondition axioms for primitive actions We represent the preconditions of the primitive actions (i.e., the conditions that make the performance of the action possible) by an axiom of the following type:

$$\forall\alpha\forall S\ Poss(\alpha, S) \leftrightarrow pre(\alpha, S) \quad (4)$$

Precondition axioms for sequence We then extend this kind of axioms for a sequence $\alpha;\beta$ where α is a primitive action and β is a complex action as follows:

$$\forall S\forall G\forall\alpha\forall\beta\ \ Poss(\alpha, S) \wedge Poss(\beta, do(G, \alpha, S)) \leftrightarrow Poss(\alpha;\beta, S) \quad (5)$$

Axiom (5) expresses that $\alpha;\beta$ is possible in S iff α is possible in S and β is possible after the performance of α in S.

Successor state axioms Following Reiter [16], for any *fluent* $f(t_1, \ldots, t_n)$, we consider a successor state axiom which specifies all the ways the value of the fluent may change.

$$\forall S\forall G\forall\alpha\ \ Poss(\alpha, S) \rightarrow f(t_1, \ldots, t_n, do(G, \alpha, S)) \leftrightarrow \quad (6)$$
$$\gamma_f^+(t_1, \ldots, t_n, \alpha, S) \vee (f(t_1, \ldots, t_n, S) \wedge \neg\gamma_f^-(t_1, \ldots, t_n, \alpha, S))$$

$\gamma_f^+(t_1, \ldots, t_n, \alpha, S)$ represents the conditions which make f true after α has been performed in S. $\gamma_f^-(t_1, \ldots, t_n, \alpha, S)$ represents the conditions which make f false after α has been performed in S.

Since *competent* is a *fluent*, we have to express a successor state axiom for it. In this paper, we assume that the competence is not deleted, i.e once an agent is competent to perform an action. This is expressed by:

$$\forall\alpha\forall\beta\forall G\forall S\ \ Poss(\beta, S) \rightarrow (competent(G, \alpha, do(G, \beta, S)) \leftrightarrow competent(G, \alpha, S)) \quad (7)$$

One can wonder why we have chosen to use a fluent to represent competence if we assume that competence does not change during execution of action. Let us claim that our modelling allows to relax this assumption easily by modifying axiom (7).

Theoretical ability axioms For any primitive action α, we consider an axiom of the following form:

$$\forall G\forall S\ \ competent(G, \alpha, S) \wedge conditions\_t(G, \alpha, S) \rightarrow able\_t(G, \alpha, S) \quad (8)$$

It expresses that a group G is theoretically able to perform α in situation S if G is competent for α in S and if some conditions related to G and α are satisfied.

Finally, in order to derive the theoretical ability for a group of agents, we consider:

$$\forall G \forall G' \forall \alpha \forall S \;\; primitive(\alpha) \wedge (G' \subseteq G) \wedge able\_t(G', \alpha, S) \rightarrow able\_t(G, \alpha, S) \quad (9)$$

$$\forall G\, G'\, \alpha\, \beta\, S \;\; (G' \subseteq G) \wedge able\_t(G', \alpha, S) \wedge able\_t(G, \beta, do(G', \alpha, S)) \rightarrow able\_t(G, \alpha;\beta, S) \quad (10)$$

Axiom (9) expresses the fact that if a sub group G' of G is theoretically able to perform a primitive action α, then the group G is also theoretically able to perform α. Axiom (10) expresses that if a sub-group G' of G is theoretically able to perform α and if G is theoretically able to perform β once G' has performed α, then G is theoretically able to perform $\alpha;\beta$ (i.e., to perform α then β).

Ability axioms Finally, the following axiom allows to derive the ability of a group:

$$\forall G \forall \alpha \forall S \; able\_t(G, \alpha, S) \wedge Poss(\alpha, S) \rightarrow able(G, \alpha, S) \quad (11)$$

4 Some properties of this model

Proposition 1. *Let $\Sigma = \{(1), \ldots, (11)\}$ be the set of axioms presented previously. Then :*

$$\Sigma \vdash \forall \alpha \forall \beta \forall G \forall G' \forall S \;\; able(G, \alpha, S) \wedge able(G', \beta, do(G, \alpha, S)) \rightarrow able(G \cup G', \alpha;\beta, S)$$

This proposition is proved by an inductive proof on the length of the sequence $\alpha;\beta$.

This proposition means that if the group G is able to perform α in the situation s and if the group G' is able to perform β after G has performed α in S, then the group $G \cup G'$ is able to perform $\alpha;\beta$ in S.

A corollary is the following:

Proposition 2. *Let $\Sigma = \{(1), \ldots, (11)\}$ be the set of axioms presented previously. Then :*

$$\Sigma \vdash \forall \alpha \forall \beta \forall G \forall S \;\; able(G, \alpha, S) \wedge able(G, \beta, do(G, \alpha, S)) \rightarrow able(G, \alpha;\beta, S)$$

Proposition 3. *Let $\Sigma = \{(1), \ldots, (11)\}$ be the set of axioms presented previously, f be a fluent and α a primitive action.*
$\Sigma \vdash \forall G \forall S \;\; Poss(\alpha, S) \rightarrow f(\ldots, do(G, \alpha, S)) \not\Longrightarrow \Sigma \vdash \forall S \forall G' \;\; f(\ldots, S) \rightarrow able(G', \alpha, S)$.

This proposition is proved by finding a counter example.

This result guaranties that we do not validate the paradox mentioned in introduction: "if φ is true, then the agent has the ability to see to it that φ". Reformulated in our model, this come to say that we do not validate: if α is an action so that a postcondition is that f is true (i.e, after any performance of α by a group of agents G, f is true) and if f is true in a situation S, then any group of agents G' is able to perform α.

5 Example

5.1 Description of the example

Let a, b and c be three agents.

- Primitive actions we consider are: to lift the door ($lift$), to sand the door ($sand$), to paint the door ($paint$).
- Competence of agents are: a is competent for sanding the door and for painting it. b is only competent for painting the door. a and b together are competent for lifting the door. c is not competent for any action.
- The initial situation is such that there is a sander ($sander(S_0)$) and it works, there is some paint and the agents are not tired (for each agent a, $ok(a, S_0)$ holds).
- Sanding is possible if the sander works.
- Painting is possible if there is some paint ($paint_r$).
- An agent is theoretically able to sand the door if it is competent for doing so and if it is not tired ; an agent is theoretically able to paint the door is it is competent for doing so and if it is not tired ; two agents are together theoretically able to lift the door is they are together competent for doing so and if they are not tired.
- Successive state axioms are defined as follows:
  An agent is tired iff it has sanded the door, or it has participated in lifting the door.
  There is paint left after the execution of an action, except if it is a painting action.
  No action makes the sander out.

5.2 Formulas in the Situation Calculus

Description of the initial situation (axioms (1) and (2))

$$ok(a, S_0)$$
$$ok(b, S_0)$$
$$ok(c, S_0)$$
$$paint\_r(S_0)$$
$$sander(S_0)$$
$$competent(\{a, b\}, lift, S_0)$$

$$competent(\{b,a\}, lift, S_0)$$
$$competent(a, sand, S_0)$$
$$competent(a, paint, S_0)$$
$$competent(b, paint, S_0)$$

Primitive actions (axioms (3))

$$primitive(lift)$$
$$primitive(sand)$$
$$primitive(paint)$$

Preconditions axioms for primitive actions (axioms (4))

$$\forall S \quad Poss(lift, S)$$

$$\forall S \quad sander(S) \leftrightarrow Poss(sand, S)$$

$$\forall S \quad paint\_r(S) \leftrightarrow Poss(paint, S)$$

Successive state axioms (axioms (6))

$$\forall B \forall Y \forall S \quad Poss(Y, S) \rightarrow (sander(do(B, Y, S)) \leftrightarrow sander(S))$$

$$\forall A \forall X \forall S \quad Poss(X, S) \rightarrow (paint\_r(do(A, X, S)) \leftrightarrow paint\_r(S) \wedge \neg(X = paint))$$

$$\begin{gathered} \forall A \forall B \forall X \forall S \quad poss(X, S) \rightarrow (ok(A, do(B, X, S)) \leftrightarrow \\ ((ok(A, S) \wedge (B = A) \wedge \neg(X = sand)) \vee \\ (ok(A, S) \wedge (A \in B) \wedge \neg(X = lift)) \vee \\ (ok(A, S) \wedge \neg(A \in B)))) \end{gathered}$$

Theoretical ability axioms (axioms (8))

$$\forall A \forall B \forall S \;\; competent(A, B, lift, S) \wedge ok(A, S) \wedge ok(B, S) \rightarrow able\_t(\{A, B\}, lift, S)$$

$$\forall A \forall S \quad competent(A, sand, S) \wedge ok(A, S) \rightarrow able\_t(A, sand, S)$$

$$\forall A \forall S \quad competent(A, paint, S) \wedge ok(A, S) \rightarrow able\_t(A, paint, S)$$

5.3 Some conclusions

Let us denote Σ the set of axioms (1),...,(11). Then,

- $\vdash \Sigma \rightarrow able\_t(a, paint, S_0)$
  In the initial situation, a is theoretically able to paint the door because it is competent for doing it and it is not tired.
- $\vdash \Sigma \rightarrow able\_t(a, paint, do(a, paint, S_0))$
  Since painting does not make the agent tired, a is still theoretically able to paint after he has painted.
- $\not\vdash \Sigma \rightarrow able(a, paint, do(a, paint, S_0))$
  But, after a has painted the door, there is no more paint, so it cannot be proved that a is able to paint the door again (even if it is theoretically able as it is shown previously)
- $\not\vdash \Sigma \rightarrow able\_t(a, sand; paint, S_0)$
  Indeed, after having sanded the door, a will be tired, so he will not be theoretically able to paint the door.
- $\not\vdash \Sigma \rightarrow able\_t(a, paint, do(\{a, b\}, lift, S_0))$
  After the group a, b has lifted the door, a and b are tired. Thus, a is not theoretically able to paint the door.
- $\vdash \Sigma \rightarrow able(\{a, b\}, paint; lift, S_0)$
  The group a, b is able to paint then lift the door. Indeed once a or b will have painted the door, a and b will not be tired. So they will be able to lift the door.
- $\not\vdash \Sigma \rightarrow able(\{a, b\}, lift; paint, S_0)$
  The group a, b is not able to lift then to paint the door. Indeed once a and b will have lifted the door, a and b will both be tired. So none of them will be able to paint the door.
- Let us add now that agent c is competent for painting the door:
  Let Σ' the set obtained by adding the formula $competent(c, paint, S_0)$ to Σ.
  Thus $\vdash \Sigma' \rightarrow able(\{a, b, c\}, lift; paint, S_0)$
  The group a, b, c is now able to lift then to paint the door. Indeed, if agents a and b lift the door, then this does not make c tired. So c is able to paint the door after a and b have lifted the door.

6 Implementation

This model has been implemented in Prolog. As in [13], we use a binary predicate `holds` in order to represent fluents. For instance, $ok(a, S_0)$ is represented by `holds(ok([a]), S0)`. The successor state axiom for fluent *ok* is expressed by the following clause:

```
holds(ok(A), do(B,X,S)) :-
   B=A, \+ (X=sand), holds(ok(A),S), Poss(X,S);
   member(A,B), \+ (X=lift), holds(ok(A),S), Poss(X,S);
   \+ member(A,B), holds(ok(A),S), Poss(X,S).
```

We can show that, given a group of agents G and an action α primitive or not, we have $\Sigma \vdash able\_t(G, \alpha, S_0)$ (resp. $\Sigma \vdash able(G, \alpha, S_0)$) if and only if Prolog with negation as failure proves `holds(able_t(G,alpha),S0)` (resp. `holds(able(G,alpha),S0)`). For instance, resuming the example presented in section 5.3 using negation as failure, we can now show that the answer to the question `holds(able([a,b], [paint,lift], S0))` is `yes` and the answer to `holds(able([a,b], [lift,paint], S0)` is `no`[3].

7 Discussion

In this paper, we have presented an attempt to model in the Situation Calculus the notions of theoretical ability and ability of an agent towards an action in a multi-agent context. Definitions of these two notions to groups of agents has also been given.

In this model, agents' theoretical ability depends on their competence and on some conditions depending on the agents. Agents' ability is then defined from theoretical ability and from some conditions which do not depend on the agents.

Introducing the notion of theoretical ability is of course interesting from a modelling point of view since it makes a distinction between conditions which are related to the agents who perform the actions and conditions which are not. But it may also be interesting in the preliminary phase of planning when choosing the agents who will be in charge of the task to be performed. Indeed, proving that the chosen agents are not even theoretically able to perform the global task is enough to prove that the task will never be performed by these agents and that changing agents is required.

However, if one is only interested in proving that a group of agents is able to perform an action, the intermediary notion of theoretical ability is not useful and definitions have to be compacted as follows:

$$\forall G \forall S \;\; competent(G, \alpha, S) \wedge conditions\_t(G, \alpha, S) \wedge Poss(\alpha, S) \rightarrow able(G, \alpha, S)$$

This preliminary work has many perspectives.

First, some more formal properties on this model must be proved. In particular, formal relations with existing works mentioned in the introduction have to be established.

Secondly, another assumption could be made when inferring the ability of a group from the abilities of its agents. Indeed, the model presented here assumes that a group of agents is able to perform an action if one of its member is able to do so. But this assumes that the conditions for an agent to be theoretically able to perform an action do not depend on the fact that this agent belongs or not to a group. But it could happen that a single agent is theoretically able to perform an action but when it belongs to a group, it is no longer able (not because the others agents prevent him to do so but because belonging to a group changes the conditions sufficient for him to be theoretically able to perform the action).

[3] Notice that we implement sequence of actions as lists in Prolog.

Thirdly, we have to extend this work by considering more types of complex actions like concurrence, iteration or conditionals. We must also take into account time and action durations. For doing so, the solution provided in [6] can be adopted.

Finally, the model presented here does not take external actions into account. In particular, fluents are changed only by actions performed by the agents we consider. But in many applications, the world may change because some other agents we don't know change it. A immediate solution we could study, consists in introducing an "external agent" who could be used to model the evolution of the world which are independent from the other agents.

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References

1. B. Chaib-Draa and R. Demolombe. L'interaction comme champ de recherche. *Information - Interaction - Intelligence, numéro spécial Modèles Formels de l'Interaction*, pages 5–24, 2001.
2. B.F. Chellas. *Modal logic. An introduction.* Cambridge University Press, 1980.
3. L. Cholvy and C. Garion. Distribution of goals addressed to a group of agents. In J. S. Rosenschein, T. Sandholm, M. Wooldridge, and M. Yokoo, editors, *Proceedings of the Second International Joint Conference on Autonomous Agents and Multiagent Systems*, pages 765–772. ACM Press, July 2003.
4. P.R. Cohen and H.J. Levesque. Intention is choice with commitment. *Artificial Intelligence*, 42:213–261, 1990.
5. R. Demolombe. Formalisation en logique des interactions entre agents : quels concepts formaliser ? Technical report, ONERA/DTIM, 2000. In French.
6. R. Demolombe and E. Hamon. What does it mean that an agent is performing a typical procedure: a formal definition in the Situation Calculus. In *Proceedings of the First International Joint Conference an Autonomous Agents and Multiagent Systems (AAMAS'02)*, pages 905–911, Bologne, 2002.
7. D. Elgesem. The modal logic of agency. *Nordic Journal of Philosophical Logic*, 2(2):1–46, 1997.
8. B.J. Grosz, L. Hunsberger, and S. Kraus. Planning and acting together. *AI Magazine*, 20(4):23–34, 1999.
9. D. Harel. Dynamic logic. In D. Gabbay and F. Guenthner, editors, *Handbook of Philosophical Logic*, volume 2, pages 497–604. D. Reidel Publishing Company, 1984.
10. J.F. Horty. Agency and obligation. *Synthese*, 108:269–307, 1996.
11. J.F. Horty and N. Belnap. The deliberative stit : a study of action, omission, ability and obligation. *Journal of Philosophical Logic*, 24:583–644, 1995. Reprinted in *The Philosopher's Annual, Volume 18-1995*, Ridgeview Publishing Company, 1997.
12. Y. Lespérance, H.J. Levesque, F. Lin, and R.B. Scherl. Ability and knowing how in the situation calculus. *Studia Logica*, 66(1):165–186, oct 2000.
13. H.J. Levesque, R. Reiter, Y. Lespérance, F. Lin, and R. Scherl. GOLOG: A logic programming language for dynamic domains. *Journal of Logic Programming*, 31:59–84, 1997.

14. J. MacCarthy and P. Hayes. Some philosophical problems from the standpoint of artificial intelligence. In D. Michie and B. Melzer, editors, *Machine Intelligence 4*, pages 463–502. Edinburgh University Press, 1969.
15. M. Pauly. A modal logic for conditional power in games. *Journal of Logic and Computation*, 12(1):149–166, 2000.
16. R. Reiter. The frame problem in the situation calculus: a simple solution (sometimes) and a completeness result for goal regression. In *Artificial Intelligence and Mathematical Theory of Computation: Papers in Honor of John McCarthy*, pages 359–380. Academic Press, New York, 1991.
17. R. Thomason. Ability, action and context. Presentation at the Temporality and Discourse Context: Dynamic and Modal Approaches Workshop, 2001.
18. B. van Linder, W. van der Hoek, and J.-J. Ch. Meyer. Formalizing abilities and opportunities of agents. *Fundamenta Informaticae*, 34(1-2):53–101, 1998.